Enhancement of the current in a superconductor strip by means of curved superconducting shields

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The distribution of the sheet current in a superconductor strip located between superconducting shields of various geometries is studied on the basis of exact analytic forms. Whereas the current becomes increasingly uniform when flat shields approach the strip, strong inhomogeneities are found to develop for curved shields, with concomitant enhancements by several times of the maximum total current predicted in the marginal Meissner state; a trait which could be favorably exploited regarding superconductor electronics applications. © 2003 American Institute of Physics. [DOI: 10.1063/1.1560866]

The sharp peaking of the current at the edges of superconductor strips is known to cause penetration of magnetic flux leading to a nonlinear electromotive response, thereby impairing the performance of superconducting devices such as filters and transmission lines. The usual arrangement which substantially mitigates the inhomogeneity of the sheet current across the width of the strip consists of a superconductor film deposited on an insulated, flat superconducting shield. At small distances between the film and the shield, the edge current peaks decrease, producing an almost homogeneous sheet current distribution. This fact is widely taken advantage of for obtaining constant sheet current profiles in shielded, long Josephson junctions too. A similar effect exists in configurations with bulk shields of soft magnetic materials of high permeability which allow one to control the spatial variation of the magnetic field, and hence, the transport current coupled to the field. Some magnetic environments of complicated geometry yield current inhomogeneities around the center of the strip, resulting in significant enhancements of the total loss-free current that can be carried by the strip. The strongly overcritical currents, recently observed in magnetically shielded superconductor strips, could be used to greatly amplify the output signal of microstrip devices, although—from a practical point of view—devices with all-superconducting components seem to be preferred. Here, therefore, we study the shielding effect in superconducting configurations more elaborate than those commonly employed.

Let us consider a flux-free, current-carrying superconductor strip of thickness $d$ and width $w$ within the range $-w/2 < x < w/2$, intersected symmetrically by the plane $y=0$ and extended infinitely in the $z$ direction of a cartesian coordinate system $x, y, z$. Assuming $d \ll w$, the variations of the current over the thickness of the strip may be ignored and, for mathematical convenience, the strip regarded as infinitesimally thin, enabling the magnetic field $H$ around the strip to be expressed through the sheet current $J$ alone. This current, in turn, is determined by the discontinuity of the tangential component of the magnetic field, $H_x$, across the surfaces of the strip for $-w/2 < x < w/2$:

$$J(x) = H_x(x, y=0+)-H_x(x, y=0-).$$  

(1)

The configuration addressed first is thought to include a pair of massive, flat superconducting shields in the Meissner state, extending across the spaces $y>a$ and $y<-a$, respectively, their boundaries occupying the planes $y = \pm a$. Aiming primarily to explore fundamental features of shielding in sophisticated geometries, London penetration into the shields is deemed negligible on the adopted macroscopic scale. In such an approach, the magnetic field lines run tangential to the various surfaces involved, precisely like the flow lines of a perfect and incompressible fluid, which is vortex-free between the strip and the shields, would do.

We represent the magnetic field under consideration as the field generated by the strip itself, supplemented with the fields due to strip images at $y = 2na$ for $n = \pm 1, \pm 2, \pm 3, \ldots$; a pattern of images which resembles that in the case of magnetic shields of the same geometry, except that the sign of the image currents here alternates for the normal component of the magnetic field, $H_x$, to vanish on the surfaces of the superconducting shields. According to Ampère's law

$$H(x,y) = \frac{1}{2\pi} \sum_{n} \frac{(-1)^{n+1}}{(2na-y,x-x')^2+(y-2na)^2} \int_{-w/2}^{w/2} dx' J(x'),$$  

(2)

where the density of the sheet current, $J_s$, acts as a planar source. The flux-free Meissner state of the strip requires that $H_x$ be zero at $y=0$. Applying this condition to Eq. (2) and performing the summation yields the following homogeneous integral equation for the sheet current distribution:

$$\int_{-w/2}^{w/2} dx' J(x') \left( \frac{1}{\sinh[\pi(x-x')/2a]} \right) = 0$$  

(3)

with the exact solution in $-w/2 < x < w/2$ appropriate to the flat shielding configuration

$$J_0(x) = J_0 \sqrt{\sinh[\pi(w+2x)/4a] \sinh[\pi(w-2x)/4a]}.$$  

(4)
The constant $J_0$ herein may be linked to the total current $I_{th}$ by integrating Eq. (4) over the width of the strip which gives,

$$J_0 = (\pi I_0/4a) \cosh(\pi w/4a)K[\tanh^2(\pi w/4a)],$$

where $K$ denotes the complete elliptic integral of the first kind.

We comment that, because of our neglect of the finite thickness of the strip, the idealized sheet current distribution, Eq. (4), diverges for $x \rightarrow \pm w/2$, whereas in reality it saturates at a distance from the order of $d$ from the edges of the strip. The Meissner state persists as long as $I_{th}$ at $x = \pm (w/2 - d)$ remains below some threshold value $J_c$ for first entry of magnetic flux. $J_c$ is determined by edge barriers of various kinds, like Bean–Livingston and geometrical barriers, or by flux pinning inside the superconductor strip which simulates an extended edge barrier itself. A geometrical barrier, for instance, yields $J_c = H_c$, where $H_c$ denotes the bulk lower critical magnetic field, and pinning gives $J_c = J_p$, where $J_p$ means the bulk critical current density. The maximum total current prevailing in the marginal Meissner state, $I_{th}$, obtains when $J_{cd}(\pm (w/2 - d)) = J_c$. The sheet current distribution applying to this state may therefore be calculated from Eq. (4) within the limits $x = \pm (w/2 - d)$ and set equal to $J_c$ beyond; the procedure adopted here.

A cross-sectional view of the flat shielding configuration for different values of the distance between the superconducting shields and the superconductor strip together with the sheet current so defined as a function of the coordinate across the strip is displayed in Fig. 1. This shows that the current distribution becomes increasingly uniform, the maximum total current growing concomitantly when the shields approach the strip.

Motivated by the findings for nonplanar magnetic shields and by the analogy between magnetic and fluid flux for superconducting shields, we study next the effect of curved superconducting environments on the distribution of the sheet current in the superconductor strip. Due to compression of magnetic flux, shielding configurations with contraction should, from Eq. (1), favor enhancements of the current in the strip. Referring to the method of conformal transformation, we therefore propose to relate the $x,y$ plane to the $\xi,\eta$ plane by

$$\xi + i \eta = c[(1 + (x + iy)/b)^p - (1 - (x - iy)/b)^p]$$

where

$$c = ((w/2)/((1 + w/2b)^p - (1 - w/2b)^p),$$

with parameters $b$ and $p$ confined to $b > w/2$ and $0 < p < 1$. While mapping the superconductor strip onto itself, Eq. (6) transforms the boundaries of the flat superconducting shields, $y = \pm a$, into curved surfaces of the same symmetry which, for given $b$, may be characterized by their minimum distance to the center of the strip, $d_0$, and by their radius of curvature at minimum distance to the center of the strip, $p_0$, i.e., by geometrically amenable quantities determining in turn, $a$ and $p$. Since the magnetic field between the strip and the shields admits a unique and invariant derivation from a scalar potential satisfying Laplace’s equation together with the complete boundary conditions in $x$ and $y$ or, respectively, $\xi$ and $\eta$, it follows by recalling Eq. (1) that the idealized sheet current distribution in $-w/2 < \xi < w/2$ appropriate to the curved shielding configuration obeys

$$J_{cd}(\xi) = J_0(x(\xi, \eta)) \frac{\partial x}{\partial \xi} \bigg|_{\eta = 0},$$

a relation which requires inverting the transform, Eq. (6), and evaluating the sheet current distribution, Eq. (4), obtained for the flat shielding configuration.

As noted before, the Meissner state survives provided that $\xi_{cd}$ at $\xi = \pm (w/2 - d)$ stays below $J_c$, and the maximum total current possible in the marginal Meissner state, $I_{th}$, results when $J_{cd}(\pm (w/2 - d)) = J_c$. The sheet current distribution pertaining to this state may, hence, be calculated from Eq. (8) within the limits $\xi = \pm (w/2 - d)$ and set equal to $J_c$ beyond; the procedure adopted here.

A cross-sectional view of the curved shielding configuration for different values of the distance between the tips of the superconducting shields and the superconductor strip on the one hand, and for different values of the radius of curvature of the tips of the superconducting shields on the other hand, together with the sheet current so defined as a function of the coordinate across the strip is shown in Figs. 2 and 3. This reveals the following traits. The current distribution becomes increasingly nonuniform, developing a central hump when the tips of the shields approach the strip; the maximum total current grows accordingly, such that an enhancement by a factor of about 2 compared with the case of flat shields results for the shortest distance set (cf. Fig. 2 with Fig. 1). The central hump of the current distribution becomes
increasingly dominant when the tips of the shields contract; the maximum total current grows accordingly, such that an enhancement by a factor of about 5 compared with the case of flat shields results for the smallest radius set (cf. Fig. 3 with Fig. 1). Sufficient it to add that, whereas the absolute magnitude of the maximum total current is upward bounded by about $wJ_f$, for flat shields, values as high as about 10 $wJ_f$ can easily be attained with curved shields, upon due choice of the geometrical parameters of their boundaries involved.

Large values of the sheet current around the center of the strip, and correspondingly large values of the sheet current near the tips of the shields, may give rise to local penetration of magnetic flux associated with a nonlinear electromagnetic response, contrary to our assumption of both the strip and the shields exhibiting the flux-free Meissner state. However, for superconductor surfaces of high quality, first entry of magnetic flux near the center of the strip or the shields, by virtue of the penetration criterion for strong type II superconducting materials, is not expected until the local sheet currents equal $J_I = (\kappa/\ln \kappa)H_{A1}$ with the Ginzburg–Landau parameter $\kappa$. This makes the mechanism of flux penetration relevant only if the maximum of the sheet current in the center of the strip or the shields is roughly $\kappa/\ln \kappa$ times the threshold value $J_I$, set at the edges of the strip, $\kappa \approx 1$ being representative of high-temperature superconductors.

Though wholly based on static derivations, our results are deemed to extend to the quasi-stationary regime as well. The enhancement of the total current by means of a pair of curved shields could therefore be favorably exploited in rf microstrip devices, where the currents are severely limited by their crowding at the edges of the strip. Moreover, the analogy with fluid flow invoked before suggests that an enhancement effect should occur in one-sided curved shielding configurations also.

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