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Thermo-electromagnetic properties of a magnetically shielded superconductor strip: theoretical foundations and numerical simulations

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Abstract
Numerical simulations of thermo-electromagnetic properties of a thin type-II superconductor strip surrounded by open cavity soft-magnetic shields and exposed to an oscillating transverse magnetic field are performed by resorting to the quasistatic approximation of a vector potential approach in conjunction with the classical description of conduction of heat. The underlying definition of the superconducting constituent makes use of an extended ‘smoothed’ Bean model of the critical state, which includes the field and temperature dependence of the induced supercurrent as well. The delineation of the magnetic shields exploits the reversible-paramagnet approximation in the Langevin form, as appropriate for magnetizations with narrow Z-type loops, and considers induced eddy currents too. The coolant is envisaged as acting like a bath that instantly takes away surplus heat. Based on the Jacobian-free Newton–Krylov approach and the backward Euler scheme, the numerical analysis at hand is tailored to the problem of a high width/thickness aspect ratio of the superconductor strip. Assigning representative materials characteristics and conditions of the applied magnetic field, the main findings for a practically relevant magnet configuration include: (i) an overall rise of the maximum temperature of the superconductor strip tending to saturation in a superconducting thermo-electromagnetic steady state above the operating temperature, magnetic shielding lending increased stability and smoothing the temperature profile along the width of the superconductor strip; (ii) a washing out of the profile of the magnetic induction and a lowering of its strength, a relaxation of the profile of the supercurrent density and an increase of its strength, a tightening of the power loss density and a reduction of its strength, all inside the superconductor strip. The hysteretic ac loss suffered by the superconductor strip is seen to be cut back or, at most, to converge on that of an unshielded strip, thermo-electromagnetic coupling merely playing an insignificant part thereby.

(Some figures may appear in colour only in the online journal)

1. Introduction

Type-II superconductors are widely used for their exceptional properties which set them off against conductors in the normal state, e.g. the ability to carry large electrical currents with only minimum hysteretic ac loss and the capacity to withstand high magnetic fields. Although these properties do not go unlimited, they can be improved by taking advantage of the shielding effect of magnetically susceptible environments. Thus, magnetically sheathed single...
or composite superconductor cables stand out for weakening the (applied and self-induced) magnetic fields experienced by their superconducting constituents [1–3]. Heterostructures made up of thin superconductor strips and paramagnetic or soft-magnetic shields of specially tailored shapes—elements of both large-scale power and microelectronic device applications deemed particularly promising—admit intricate control of the supercurrent density and the magnetic flux [4–12]. Lowering the flux density near the edges of such strips entails a distinct mitigation of the influence of the self-induced magnetic field exerted upon the strips [13].

The idea of guiding the magnetic flux with the help of magnetic shields in order to augment superconductor performance has been advanced or substantiated through theoretical predictions [4, 5, 14–20] and experimental facts [1, 21–23]. Analysis of the distributions of the supercurrent density and the magnetic field provided the key to understanding the principles of the magnetic shielding effect [1, 4, 5, 24, 25]; investigations devoted to its optimization gave hints for suitably shaping and positioning the magnetic shields [13, 26–31]. However, all these studies of the distributions of the electrical current and the magnetic flux as well as of hysteretic ac losses in superconductors with magnetic shields have been performed on purely electromagnetic grounds, omitting the conversion of electromagnetic energy into Joule heat. The latter gives rise to a local increase of the superconductor temperature and therefore entails a reduction of the critical current which, in turn, boosts the production of Joule heat, followed by a further increase of temperature. Such a vicious circle would continue until quench and the transition to the normal state, unless the removal of surplus heat from the superconductor strip was efficient enough to permit the establishment of a superconducting thermo-electromagnetic steady state. Thus, it is indispensable to also allow for thermal aspects in the electromagnetic modelling of superconductor/soft-magnet heterostructures, if issues like thermal stability and the influence of temperature on their electromagnetic properties themselves are to be addressed [32].

Thermo-electromagnetic behaviour of superconductors has been examined, both analytically and numerically, in the case of various settings before: a bulk superconductor exposed to an applied magnetic field [33] or levitated above a magnetic track [34, 35], an arrangement of two superconductor slabs with a transport current flowing inside [36], and coated conductors of different geometries subject to a fault current wave [37, 38]. Other investigations concerned the normal zone propagation characteristics, i.e. quench, in coated high-aspect ratio superconducting tapes, films and coils [39–42]. Hysteric ac losses in superconducting coils with magnetic or non-magnetic substrates have been estimated too [43]. Thermal properties of thin superconductor strips due to the motion of magnetic vortices have been appraised, last but not least [44, 45].

Motivated by the opportunities for guiding the magnetic flux and by the necessity to include the action of the cryogenic part, developing further previous investigations on purely electromagnetic grounds, we here study thermo-electromagnetic properties of a magnetically shielded thin type-II superconductor strip subject to an oscillating transverse magnetic field, aiming to get deeper insight into the performance of this type of superconductor/soft-magnet heterostructure constituent. Calling upon the quasistatic approximation of a vector potential approach together with the classical description of conduction of heat, in section 2 we define the theoretical model of the liquid-cooled superconductor strip surrounded by soft-magnetic shields and set out the general framework which lays the basis of our numerical simulations, studying two essentially different magnet configurations—a horseshoe-shaped magnetic environment representing a concave open cavity and a wedge-shaped magnetic environment representing a convex open cavity—expected to yield particularly strong shielding effects. Making recourse to these, in section 3 we present and discuss the numerical results procured for the initial rise of the maximum temperature of the superconductor strip and the stability of its thermo-electromagnetic steady state. Selecting the magnet configuration with superior performance of the configurations addressed, we show the numerical results displayed for the transient evolution of the temperature profile along the width of the strip. Turning to the thermo-electromagnetic steady state, we portray the distributions of the magnetic induction, the supercurrent density and the power loss density inside the superconductor strip and also delineate the hysteretic ac loss suffered by the strip. Finally, in section 4 we conclude by summarizing and highlighting the results attained. Details of the hysteretic ac loss of a thermo-electromagnetically isolated superconductor strip, putting our investigations into an established perspective, can be found in the appendix.

2. Theoretical model

Let us consider an infinitely extended type-II superconductor strip of width \(2w\) and thickness \(d \ll w\) limited by the range \(-w \leq x \leq w\) and located between two infinitely extended, homogeneous soft magnets specifying a horseshoe-shaped magnet configuration or a wedge-shaped magnet configuration, the direction of translational invariance of these superconductor/soft-magnet heterostructures being parallel to the \(z\)-axis of a Cartesian coordinate system \(x, y, z\), as depicted in figure 1. The choice of their geometrical design is motivated by previous realizations that open magnetic environments yield particularly strong shielding effects [4, 5, 13]. A cross section distinguishes the domain occupied by the strip from the respective domains occupied by the soft magnets, leaving the residual domain for a cooling liquid to pervade the space in between. We understand that the strip is subject to an oscillating transverse magnetic field, without a transport current flowing inside.

For premises of this sort, the magnetic field \(\mathbf{H}\), the magnetization \(\mathbf{M}\) and the magnetic induction \(\mathbf{B}\), related by

\[
\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})
\]

with the permeability of a vacuum \(\mu_0\), only reveal non-zero \(x\)- and \(y\)-components; the electric field \(\mathbf{E}\), the electrical current
as well as time 
which we subject to the Coulomb gauge 
induction with equation (2), the non-zero components of the magnetic 
temperature 
physical state of the strip therefore is fully characterized by 
involving a single nontrivial electromagnetic quantity, 

\[ A \]

magnetic induction \( B \)

density \( j \) and the magnetic vector potential \( A \) linked to the magnetic induction \( B \) via 

\[ B = \nabla \times A, \]

however, only display a non-zero \( z \)-component. It is thus expedient to adopt the vector potential representation merely involving a single nontrivial electromagnetic quantity, \( A_z \), which we subject to the Coulomb gauge \( \partial A_z / \partial z = 0 \). The physical state of the strip therefore is fully characterized by the magnetic vector potential component \( A_z \), together with temperature \( T \), both depending on the coordinates \( x \) and \( y \) as well as time \( t \). From Faraday’s law in conjunction with equation (2), the non-zero components of the magnetic induction \( B \) and the electric field \( E \) then follow as 

\[ B_y = \frac{\partial A_z}{\partial y}, \quad B_x = \frac{\partial A_z}{\partial x}, \quad \frac{\partial A_z}{\partial t} = -E_z \]

alongside the distribution of temperature \( T \).

2.1. Governing equations

We first present the master equations for the two state variables \( A_z \) and \( T \) in a general form, before specializing with respect to the various domains and identifying the supplementary conditions that our theoretical model underlies.

Turning to the Maxwell equations in the quasistatic approximation and exploiting equations (1) and (2), we formally arrive at the electromagnetic master equation for \( A_z \),

\[ \nu_0 \left( \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} \right) + \frac{\partial M_x}{\partial x} - \frac{\partial M_y}{\partial y} = -j_z, \]

where \( \nu_0 = 1/\mu_0 \) denotes the magnetic reluctance of a vacuum. Equation (4) requires identification of a magnetization law and substitution for the spatial derivatives \( \partial A_z / \partial x \) and \( \partial A_z / \partial y \); it further necessitates the choice of a current-field relation connecting \( j_z \) with \( E_z \), i.e. with the temporal derivative \( \partial A_z / \partial t \), from equation (3). A solution of equation (4) thus allows the electromagnetic power loss density,

\[ Q = j_z E_z, \]

to be deduced. This, on the other hand, admits the hysteretic ac loss suffered during a cycle of the applied magnetic field, per unit length in the \( z \)-direction,

\[ U_{ac} = \oint \text{d}t \oint \Omega \text{d}x \text{d}y Q, \]

to be derived; the integration thereby extends over the full cycle and the cross section \( \Omega \) of the respective domain. Taking the classical view of conduction of heat in a homogeneous isotropic solid, we end up with the thermal master equation for \( T \),

\[ \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - c \left( \frac{\partial T}{\partial t} \right) = -Q, \]

the thermal conductivity \( \kappa \) and the heat capacity per unit volume \( c \) herein being independent of the coordinates of space; the source term, given by equation (5), arises from the conversion of electromagnetic energy into Joule heat.

Equations (4) and (7) make clear that changes with respect to space and time of the electromagnetic power loss cause spatio-temporal variations of temperature; these, in turn, act back upon the electromagnetic observables themselves through their dependence on temperature—a manifestation of self-consistency ensured by the simultaneous solution of equations (4) and (7) in the various domains.

2.1.1. Superconductor domain. We assume that magnetization of the superconductor strip is virtually absent and the constitutive relation for the \( z \)-component of the induced supercurrent density \( j_z \) is well represented by the ‘smoothed’ Bean model of the critical state in the hyperbolic–tangent approximation,

\[ j_z = j_c \tanh(E_c/E_0), \]

as measurements confirm [46, 47], where the auxiliary electric field \( E_0 \) relates to the critical current density \( j_{c0} \) and the electrical conductivity \( \sigma_0 \) at operating temperature/zero field via \( E_0 = j_{c0}/\sigma_0 \). Equation (8) describes a continuous change of \( j_z \) between virtually \( -j_c \) and virtually \( j_c \) across a transitional range of the \( z \)-component of the electric field of extent \( |\Delta E_z| \cong 3E_0 \) about \( E_z = 0 \), when \( E_z \) alters sign. (Bean’s idealized model [48, 49], implying a discontinuous change between \( -j_{c0} \) and \( j_{c0} \) in the absence of a transitional range, recovers from equation (8) on letting \( j_c \rightarrow j_{c0} \), \( \sigma_0 \rightarrow \infty \), and hence \( E_0 \rightarrow 0 \).) Making use of equations (3) and (8) in equation (4), we therefore procure the electromagnetic master equation for \( A_z \),

\[ \nu_0 \left( \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} \right) = j_c \tanh \left( \frac{\partial A_z}{\partial t} / E_0 \right). \]
If the amplitude of the induced supercurrent density \( j_c \) is considered to depend on the acting field within Kim’s extended ansatz for the critical state [50] and also to vary linearly with temperature [36], the representation

\[
j_c = j_{c0} \left( \frac{B_0}{B_0 + B} \right) \left( \frac{T_c - T}{T_c - T_0} \right)
\]

ensues, with the reference value of the magnetic induction \( B_0 \), the critical temperature \( T_c \) and the operating temperature \( T_0 \). Equation (2) tells that

\[
B = \sqrt{\left( \frac{\partial A_z}{\partial x} \right)^2 + \left( \frac{\partial A_z}{\partial y} \right)^2}
\]

in equation (10). Conceiving, for simplicity, the thermal conductivity and the heat capacity per unit volume in equation (7) to be adequately described by their respective values \( \kappa_0 \) and \( c_0 \) taken at the operating temperature \( T_0 \) and using equations (3), (5) and (8), we find the thermal master equation for \( T \),

\[
\kappa_0 \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = c_0 \left( \frac{\partial T}{\partial t} \right)
\]

\[
= -j_c \left( \frac{\partial A_z}{\partial t} \right) \tanh \left( \frac{\partial A_z}{\partial t}/E_0 \right),
\]

with definitions given by equations (10) and (11).

2.1.2. Soft-magnet domains. We envisage that the two soft magnets shielding the superconductor strip are made of identical materials whose magnetization demonstrates isotropy with respect to the \( x, y \)-planes of the coordinate system remarked on above. Equations (1) and (2) then readily show that

\[
\frac{\partial M_z}{\partial x} - \frac{\partial M_z}{\partial y} = -\nu_0 \left( \frac{M}{M + H} \right) \left( \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} \right),
\]

putting forward the magnitudes of the magnetization and the magnetic field, \( M \) and \( H \). We further assume that the \( z \)-component of the induced eddy current density \( j_z \) is well described by Ohm’s law,

\[
j_z = \sigma_m E_z,
\]

introducing the electrical conductivity \( \sigma_m \). Making recourse to equations (3), (13) and (14) in equation (4), we thus obtain the electromagnetic master equation for \( A_z \),

\[
\nu_m \left( \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} \right) = \sigma_m \left( \frac{\partial A_z}{\partial t} \right),
\]

with the magnetic reluctance of the soft magnets

\[
\nu_m = \nu_0 \left( \frac{H}{H + M} \right),
\]

in which

\[
H + M = \nu_0 \left[ \left( \frac{\partial A_z}{\partial x} \right)^2 + \left( \frac{\partial A_z}{\partial y} \right)^2 \right].
\]

referring to equations (1) and (2) again. If the prevailing magnetization shows a narrow Z-type loop that permits neglect of coercivity, calling upon the reversible-paramagnet approximation in the Langevin form,

\[
M = M_s [\coth(H/H_0) - H_0/H].
\]

with the saturation magnetization \( M_s \) and the auxiliary magnetic field \( H_0 \) linked to the magnetic susceptibility at zero field \( \chi_0 \) by \( H_0 = M_s/3\chi_0 \), is suggested by experiments [51–53]. We note from equation (17) that \( A_z \) determines the sum \( H + M \) and, in conjunction with equation (18), also the strength of the magnetic field \( H \). Equation (16), on the other hand, shows that \( A_z \) also governs the magnetic reluctance \( \nu_m \) which, in turn, constitutes equation (15). The latter equation together with equations (16)–(18) thus represents a problem of self-consistency in itself. Production of heat caused by eddy currents, and hence transport of heat too, are regarded as negligible, so the temperature \( T \) at any time \( t \geq 0 \) is simply given by the operating temperature \( T_0 \) throughout the soft-magnet domains.

2.1.3. Coolant domain. With both magnetization and current flow absent, equation (4) reduces to the electromagnetic master equation for \( A_z \),

\[
\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = 0.
\]

If, as believed for practical convenience, the cooling liquid were acting like a bath that instantly takes away surplus heat, the temperature \( T \) at any time \( t \geq 0 \) is again simply given by the operating temperature \( T_0 \) throughout the coolant domain.

At time \( t = 0 \), we supplement the electromagnetic master equations for \( A_z \) defined in the superconductor domain, the soft-magnet domains and the coolant domain, equations (9), (15) and (19), with the initial condition

\[
A_z = 0.
\]

Likewise, we add to the thermal master equation for \( T \) defined in the superconductor domain, equation (12), the initial condition

\[
T = T_0.
\]

At any time \( t \geq 0 \), we require that the \( z \)-component of the magnetic vector potential and its normal derivative

\[
A_z \quad \text{and} \quad \frac{\partial A_z}{\partial n}
\]

be continuous (22) when traversing the soft-magnet/coolant interfaces or the superconductor/coolant interface. For the latter boundary we furthermore demand that the condition on temperature [32]

\[
\kappa_0 \left( \frac{\partial T}{\partial n} \right) + h(T - T_0) = 0
\]

holds, with the normal derivative directed from the superconductor to the coolant domain, the surface heat transfer
coefficient $h$ herein allowing for natural convection/nucleate boiling and, if need be, transition boiling as well as film boiling in diverse ranges of the increment of temperature $T - T_0$; a straightforward representation confined to values of $T$ closely above the operating (boiling) temperature $T_0$ is

$$h = \alpha(T - T_0),$$

with the prefactor $\alpha$ assumed constant [54]. The imposition of an oscillating transverse magnetic field with amplitude $H_x$ and frequency $\nu$, finally, is modelled by the asymptotic condition that far away from the heterostructure,

$$A_z \rightarrow -\mu_0 H_x \sin(2\pi \nu t),$$

at any time $t \geq 0$.

2.2. Computational implementation

Our numerical simulations of thermo-electromagnetic properties rest upon a purposive approach tailored to the problem of a high, but finite, width/thickness aspect ratio of the superconductor strip; they run as follows. The first step includes a geometrical definition of the domains occupied by the strip and by the soft magnets making up the heterostructures. The total domain envisaged for numerical analysis, which embraces the rectangular domain of the strip as its central constituent, is outwardly delineated by a quadratic frame well encompassing the domains covered by the heterostructures. The second step involves the generation of an adapted seed of mesh points of a triangular grid, fostered by the flexibility in putting computational elements wherever they seem fit. Thus, a substantially refined mesh size $m$ near the surface and the edges of the strip, i.e. in regions with pronounced field inhomogeneities bound to occur, recommends itself; a practically still easily manageable lower limit represents $m/w = 1.0 \times 10^{-4}$ in conjunction with $d = 50m$. The third step concerns the algorithmic solution of the electromagnetic master equations for $A_z$ defined in the superconductor domain, the soft-magnet domains and the coolant domain, equations (9), (15) and (19), simultaneously with the thermal master equation for $T$ defined in the superconductor domain, equation (12). This assumes that the initial conditions, equations (20) and (21), and the boundary conditions, equations (22) and (23), as well as the far-field condition, equation (25), apply. Satisfaction of the latter, asymptotic requirement for a quadratic frame of large, but finite extent is approximately ensured by confining it to this frame’s outer boundary.

Supported by previous experience [34], discretization of space using the finite-element technique is performed taking Galerkin’s method [55] into account, whereas discretization of time is carried on the basis of the finite-difference technique via the backward Euler scheme [56]. It is worth emphasizing, however, that a main challenge to attaining fast numerical convergence of the solutions of the finite-element equations earned by discretizing the master equations for $A_z$ and $T$ does arise from the drastic change of $j_z$, equation (8), around zero $E_z$. We therefore resort to an advanced tool, the Jacobian-free Newton–Krylov approach, founded on a synergistic combination of Newton-type methods for superlinearly convergent solutions of nonlinear equations and Krylov subspace methods for solving the Newton correction equations [57]. This is used for handling the nonlinearities of the finite-element equations and solving the associated algebraic equations after linearization by means of the generalized minimal residual algorithm [58]. An invaluable advantage lies in the fact that such a course avoids the usual evaluation of the Jacobian matrix for each element, and hence saves massive demands on computer memory and processing time.

Naturally, all computer simulations are subject to tests for convergence of the predicted results with enlargements of the quadratic frame underlying the numerical analysis and refinements of the size of the meshes setting up the triangular grid; they enjoy a reduction to a half of the true computational expenditure by taking the mirror symmetry of the respective problems into account. We comment that, in the case of a quadratic frame with sides $100w$ and a representative example of 25 000 finite-element nodes meshing the space outside the domain of the strip, the prescribed relative error tolerance $\varepsilon = 1.0 \times 10^{-4}$ for solving the finite-element equations is certainly observed, and the number of iterations required at a given instant of time is mostly less than 5. The processing time called for generating an update of the spatial distribution of a physical observable with the time step of 0.2 ms is typically around 10 s on an Intel Core i7-2600 processor-driven desktop computer running at the clock speed of 3.4 GHz.

3. Results and discussion

To appraise the effect of magnetic shielding on thermo-electromagnetic properties of the superconductor strip in the presence of an oscillating transverse magnetic field, numerical simulations were carried out for the horseshoe-shaped magnet configuration and the wedge-shaped magnet configuration of figure 1, understanding that the strip is made of an yttrium–barium cuprate and the soft magnets consist of an iron- or nickel-based alloy, with liquid nitrogen serving as the coolant at the operating temperature $T_0 = 77\ K$. The amplitude and frequency of the applied magnetic field were taken as $H_x = 50\ A\ mm^{-1}$ and $\nu = 50\ Hz$, respectively, unless stated otherwise. If the width of the strip $2w$ and its thickness $d$ are fixed, the distance between the edges of the strip and the magnets $a$ remains as the only variable structural quantity; we adopted the value $a/w = 0.2$, deemed a representative choice. Our numerical evaluations, shown graphically below, relate to the geometrical and materials data listed in table 1.

3.1. Evolution of temperature

Figure 2 portrays the initial variation with time $t$ of the maximum temperature $T_{\text{max}}$ recorded on the magnetically shielded superconductor strip for the horseshoe-shaped magnet configuration and the wedge-shaped magnet configuration of figure 1, together with predictions for the limiting case of an unshielded strip. A general trait revealed for each magnet configuration is that, starting from the operating temperature...
The superconductor strip, whereby the wedge-shaped magnet configuration of figure 1(b) outperforms the horseshoe-shaped magnet configuration of figure 1(a), as far as the mitigation of the rise of the maximum temperature $T_{\text{max}}$ is concerned. This parallels the previous finding of a purely electromagnetic trait, viz. the superiority of the wedge-shaped magnet configuration over the horseshoe-shaped magnet configuration regarding the enhancement of the current-carrying capability of the superconductor strip [13]. We therefore confine ourselves to the wedge-shaped magnet configuration in studying further thermo-electromagnetic properties of the superconductor strip.

It may be commented that the increase of the maximum temperature $T_{\text{max}}$ could, to a certain extent, be further suppressed internally by reinforcing the magnetic shielding effect, either through an increase of the saturation magnetization of the soft magnets $M_s$ beyond the value implied in table 1 or through a reduction of the normalized distance between the edges of the strip and the magnets $a/w$ below the value reported above. Conversely, a decrease of the amplitude of the external magnetic field $H_b$ below the strength stated above would impede the rise of the maximum temperature $T_{\text{max}}$ and diminish the amplitude of its oscillations further still, simultaneously extending the time taken to reach a thermo-electromagnetic steady state and amplifying the magnetic shielding effect, whereas an increase of the amplitude of the external magnetic field $H_b$ beyond the strength stated above would promote the rise of the maximum temperature $T_{\text{max}}$ and enlarge the amplitude of its oscillations, ultimately leading to a quench of the magnetic shielding effect in the saturation regime of the magnetic shields.

Figure 3 depicts the variation of temperature $T$ over the width (in the virtual absence of any change across the thickness) of the magnetically shielded superconductor strip at early instants of time $t$, addressing the wedge-shaped magnet configuration of figure 1(b) including the extreme case of an unshielded strip. A general trait due to local production and transfer of heat, disclosed for a given time $t$, is that temperature $T$ shows a minimum in the centre of the strip and transfer of heat, disclosed for a given time $t$, is that temperature $T$ shows a minimum in the centre of the strip.

Data relating to the superconductor strip

- Half-width, $w$ (mm) 2 [20]
- Thickness, $d$ (μm) 10 [46]
- Critical temperature, $T_c$ (K) 92 [59]
- Critical current density at operating temperature/zero field, $j_{00}$ (A m⁻²) $2.5 \times 10^4$ [46]
- Electrical conductivity at operating temperature/zero field, $\sigma_0$ (S m⁻¹) $5.0 \times 10^4$ [46]
- Reference value of the magnetic induction, $B_0$ (T) 0.1 [60]
- Thermal conductivity, $k_0$ (W K⁻¹ m⁻¹) 7 [37]
- Heat capacity per unit volume, $c_0$ (J K⁻¹ m⁻³) $1.6 \times 10^6$ [37]

Data relating to the soft-magnetic shields

- Saturation magnetization, $M_s$ (A m⁻¹) 7.5 $\times 10^2$ [52]
- Magnetic susceptibility at zero field, $\chi_0$ 250 [52]
- Electrical conductivity, $\sigma_0$ (S m⁻¹) 2.0 $\times 10^4$ [61]

Data relating to the cooling liquid

- Prefactor of the surface heat transfer coefficient, $\alpha$ (W K⁻¹ m⁻²) 21.9 [54]
the strip. While raising its magnitude as time $t$ augments, the profile of temperature $T$ first steepens, then flattens again as a result of backflow of heat along the width of the strip when approaching the thermo-electromagnetic steady state before the end of 500 ms, the ultimate time considered in this simulation. A comparison between figures 3(a) and (b) reveals that, in addition to mitigating the increase of temperature $T$, magnetic shielding curtails the gradients of temperature $T$, and therefore smooths the temperature profile along the width of the strip.

### 3.2. Distribution of the magnetic induction

Figure 4 graphically displays the distribution of the magnetic induction $B$ inside the magnetically shielded superconductor strip at different instants of time $t$ during a full cycle of the applied magnetic field in the thermo-electromagnetic steady state, addressing the wedge-shaped magnet configuration of figure 1(b) including the extreme case of an unshielded strip. The following properties of this distribution stand out: symmetry about the mirror planes $x = 0$ and $y = 0$ of the superconductor strip as well as invariance against translations by a half-cycle of the external magnetic field. At time $t = 480$ ms, i.e., a new cycle start, the halftones of the magnetic induction $B$ represent the distribution of trapped magnetic field after excitation of the superconductor strip during the cycles before, their intensity decreasing from the centre towards the edges of the strip. At time $t = 482.5$ ms, the applied magnetic field, directed opposite to the trapped magnetic field, begins to penetrate into the superconductor strip from its marginal parts, and hence impairs the magnetic induction $B$ around the centre of the strip. At time $t = 485$ ms, when the applied magnetic field has acquired full strength, the magnetic induction $B$ demonstrates an increased magnitude throughout the domain of the superconductor strip, as compared to the situation before. At time $t = 487.5$ ms, when the applied magnetic field has become weaker again and supercurrents of opposite signs have been induced from the edges of the strip, the magnetic induction $B$ is still further reduced in the marginal parts of the strip. At time $t = 490$ ms, the original distribution of the magnetic induction $B$ with trapped magnetic field is restored, whereupon the scenario repeats. A comparison between figures 4(a) and (b) shows that, as a result of the redistribution of the magnetic field, magnetic shielding in the presence of thermo-electromagnetic coupling washes out the profile of the magnetic induction $B$ and lowers its strength, except for the phases with trapped magnetic field.

### 3.3. Distribution of the supercurrent density

Figure 5 graphically displays the distribution of the supercurrent density component $j_z$ inside the magnetically shielded superconductor strip at different instants of time $t$ during a full cycle of the applied magnetic field in the thermo-electromagnetic steady state, addressing the wedge-shaped magnet configuration of figure 1(b) including the extreme case of an unshielded strip. The following properties of this distribution stand out: antisymmetry about the mirror plane $x = 0$, and symmetry about the mirror plane $y = 0$, of the superconductor strip as well as antisymmetry with respect to translations by a half-cycle of the external magnetic field. At time $t = 480$ ms, i.e., a new cycle start, the halftones of the supercurrent density component $j_z$ bear witness to induced supercurrents of opposite signs at work in the partly flux-filled critical state, generating the trapped magnetic field. At times $t = 482.5$ ms and $t = 485$ ms, when the applied magnetic field has come into play and, respectively, obtained full strength, the supercurrent density component $j_z$ manifests an almost fully developed critical state, with homogeneous supercurrents flowing in two adjacent half-domains of the superconductor strip. At time $t = 487.5$ ms, when supercurrents of opposite signs have been induced from the edges of the strip due to the decrease of the applied magnetic field and a transition from one critical state to another has taken place, the supercurrent density
Figure 4. Distribution of the magnetic induction $B$ (unit T) inside the magnetically shielded superconductor strip at different instants of time $t$ during a cycle of an applied transverse magnetic field with amplitude $H_a = 50$ A mm$^{-1}$ and frequency $\nu = 50$ Hz in the thermo-electromagnetic steady state, referring to the wedge-shaped magnet configuration of figure 1(b) with (a) $a/w = 0.2$ and (b) $a/w \rightarrow \infty$, as for an electromagnetically isolated strip. The scale of the thickness of the strip is enlarged by the factor 100.

Component $j_z$ shows a reversal of signs with respect to the situation before and a protracted structure around the centre of the strip. At time $t = 490$ ms, the original distribution of the supercurrent density component $j_z$ in the partly flux-filled critical state is restored, save for an inflection about the mirror plane $x = 0$ of the superconductor strip, whereupon the scenario repeats. A comparison between figures 5(a) and (b) reveals that, by virtue of the induction of supercurrents of opposite signs, magnetic shielding in the presence of thermo-electromagnetic coupling can relax the profile of the supercurrent density component $j_z$ and increase its strength, not just for the phases with trapped magnetic field.
Figure 5. Distribution of the supercurrent density component $j_z$ (unit A m$^{-2}$) inside the magnetically shielded superconductor strip at different instants of time $t$ during a cycle of an applied transverse magnetic field with amplitude $H_a = 50$ A mm$^{-1}$ and frequency $\nu = 50$ Hz in the thermo-electromagnetic steady state, referring to the wedge-shaped magnet configuration of figure 1(b) with (a) $a/w = 0.2$ and (b) $a/w \to \infty$, as for an electromagnetically isolated strip. The scale of the thickness of the strip is enlarged by the factor 100.

3.4. Distribution of the power loss density

Figure 6 graphically displays the distribution of the power loss density $Q$ inside the magnetically shielded superconductor strip at different instants of time $t$ during a full cycle of the applied magnetic field in the thermo-electromagnetic steady state, addressing the wedge-shaped magnet configuration of figure 1(b) including the extreme case of an unshielded strip. The following properties of this distribution stand out: symmetry about the mirror planes $x = 0$ and $y = 0$ of the superconductor strip as well as invariance against translations by a half-cycle of the external magnetic field. At time $t = 480$ ms, i.e. a new cycle start, the halftones of the power loss
Figure 6. Distribution of the electromagnetic power loss density $Q$ (unit W m$^{-3}$) inside the magnetically shielded superconductor strip at different instants of time $t$ during a cycle of an applied transverse magnetic field with amplitude $H_a = 50$ A mm$^{-1}$ and frequency $\nu = 50$ Hz in the thermo-electromagnetic steady state, referring to the wedge-shaped magnet configuration of figure 1(b) with (a) $a/w = 0.2$ and (b) $a/w \to \infty$, as for an electromagnetically isolated strip. The scale of the thickness of the strip is enlarged by the factor 100.

density $Q$ essentially reflect variations of the magnitude of the induced electric field appearing because of the temporal change of the magnetic induction in the partly flux-filled critical state: there is a narrow minimum of the power loss density $Q$ around the centre of the strip (due to the restricted graphical resolution marked zero here), followed by an increase up to a respective maximum in the marginal parts of the strip. At times $t = 482.5$ ms and $t = 485$ ms, when the applied magnetic field has come into play and, respectively, obtained full strength, the power loss density $Q$ still reveals the qualitative traits of the situation before, yet with the overall magnitude curtailed and the extent of the minimum narrowed.
further down. At time $t = 487.5\text{ ms}$, when the transition from one critical state to another has taken place, the power loss density $Q$ shows a considerably broadened central minimum (due to the restricted graphical resolution again marked zero here) along with a substantial increase around the edges of the strip, as compared to the situation before. At time $t = 490\text{ ms}$, the original distribution of the power loss density $Q$ in the partly flux-filled critical state is restored, whereupon the scenario repeats. A comparison between figures 6(a) and (b) unveils that, by virtue of the induction of an electric field, magnetic shielding in the presence of thermo-electromagnetic coupling can tighten the profile of the power loss density $Q$ and reduce its strength, not just for the phase change of critical states.

3.5. Hysteretic ac loss

Figure 7 illustrates the dependence of the normalized hysteretic ac loss $U_{ac}/H_a^2$ on $H_a/H_c$, the normalized amplitude of the magnetic field applied to the magnetically shielded superconductor strip in the thermo-electromagnetic steady state, addressing the wedge-shaped magnet configuration of figure 1(b), together with the prediction for the limiting case of an infinitely thin strip. This introduces the characteristic magnetic field $H_c$, which may be expressed as $H_c = J_c/\pi$ in terms of the critical sheet current of an infinitesimally thin superconductor strip $J_c = d_0 \varepsilon_0$ [19, 20], yielding the value $H_c \approx 8\text{ A mm}^{-1}$ from the geometrical and materials data of table 1. A general trait revealed for both the shielded and the unshielded strip is that, starting from the smallest value of the normalized amplitude of the applied magnetic field $H_a/H_c$, a monotonic rise of the normalized hysteretic ac loss $U_{ac}/H_a^2$ towards a maximum occurs, followed by an asymptotically converging descent, as $H_a/H_c$ augments. Important quantitative differences, however, exist: apart from large values of $H_a/H_c$ defining the saturation regime of the magnetic shields where the two predictions virtually meet, the normalized hysteretic ac loss $U_{ac}/H_a^2$ is always lower for the shielded than for the unshielded strip, the effect being particularly distinct for moderate and small values of $H_a/H_c$. Moreover, the profile of the normalized hysteretic ac loss $U_{ac}/H_a^2$ for the magnetically shielded superconductor strip is tighter, its maximum—shifted towards an increased value of $H_a/H_c$—being steeper and cut back in height compared to that of an unshielded strip. Magnetic shielding thus leads to an overall improved performance of the superconductor strip.

We comment that taking electromagnetic and thermal features simultaneously into account does not bring about significant modifications of the theoretical predictions of the normalized hysteretic ac loss $U_{ac}/H_a^2$ based on a purely electromagnetic approach: the mitigating influence of thermo-electromagnetic coupling is only minute in the whole range of values of the normalized amplitude of the applied magnetic field $H_a/H_c$ displayed (being practically absent in its lower part due to the only little production of heat) and therefore hard to graphically represent. The rapid fall of the normalized hysteretic ac loss $U_{ac}/H_a^2$ at large values of $H_a/H_c$, when magnetic shielding is almost quenched, must consequently be attributed to Kim’s extended ansatz for the critical state, as calling in of figure A.1 suggests.

4. Summary and conclusion

By resorting to the quasistatic approximation of a vector potential approach in conjunction with the classical description of conduction of heat, we have investigated numerically thermo-electromagnetic properties of a thin type-II superconductor strip surrounded by the soft-magnetic shields of a horseshoe-shaped magnet configuration or a wedge-shaped magnet configuration and exposed to an oscillating transverse magnetic field. The underlying definition of the superconducting constituent makes use of an extended ‘smoothed’ Bean model of the critical state, which includes the field and temperature dependence of the induced supercurrent as well. The delineation of the magnetic shields exploits the reversible-paramagnet approximation in the Langevin form, as appropriate for magnetizations with narrow $Z$-type loops, and considers induced eddy currents too. The coolant is envisaged as acting like a bath that instantly takes away surplus heat. On the basis of the Jacobian-free Newton–Krylov approach and the backward Euler scheme, the numerical analysis at hand is tailored to the problem of a high width/thickness aspect ratio of the superconductor strip.

Numerical results for thermo-electromagnetic properties of the magnetically shielded superconductor strip have been obtained assigning representative materials characteristics and conditions of the applied magnetic field. The main findings can be summarized as follows.

(i) **Thermal transient state.** Starting from the operating temperature $T_0$, an overall rise of the maximum temperature $T_{max}$ tending to saturation in a superconducting
thermo-electromagnetic steady state occurs, modulated by oscillations of twice the frequency of the applied magnetic field. The effect of magnetic shielding here is twofold at least: to mitigate the rise of the maximum temperature $T_{\text{max}}$, and hence to lend increased stability to the thermo-electromagnetic steady state of the superconductor strip—note that the wedge-shaped magnet configuration outperforms the horseshoe-shaped magnet configuration thereby; to curtail the gradients of temperature $T$ itself, and therefore to smooth the temperature profile along the width of the strip.

(ii) Thermo-electromagnetic steady state. Magnetic shielding can also have a profound influence on electromagnetic observables of the superconductor strip: to wash out the profile of the magnetic induction $B$ and lower its strength, except for the phases with trapped magnetic field; to relax the profile of the supercurrent density component $j_z$ and increase its strength, not just for the phases with trapped magnetic field; to tighten the profile of the power loss density $Q$ and reduce its strength, not just for the phase change of critical states. Furthermore, magnetic shielding is seen to cut back the normalized hysteretic ac loss $U_{\text{ac}}/H_c^2$ and thus lead to an overall improved performance of the superconductor strip, thermo-electromagnetic coupling merely playing an insignificant part thereby.

In conclusion, a self-consistent approach such as the one described above, with its simultaneous regard of electromagnetic and thermal traits, is called for if the time taken to reach the thermo-electromagnetic steady state as well as the stability of this state, apart from the transient evolution of the temperature profile along the width of the magnetically shielded superconductor strip, is to be addressed. On the other hand, a limitation to a purely electromagnetic approach, neglecting the production of heat, seems sufficient for estimating the hysteretic ac loss suffered by the strip in the thermo-electromagnetic steady state. The computational method developed as it stands is easy to handle and ready to use, without the need for adaptations of existing codes in multi-physics software packages [62–64].

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Appendix. Thermo-electromagnetically isolated superconductor strip

We study a thermo-electromagnetically isolated type-II superconductor strip of width $2w$ and thickness $d \ll w$ limited by the range $-w \leq x \leq w$, which extends infinitely along the $z$-axis of the Cartesian coordinate system $x, y, z$, understanding that the strip is subject to an oscillating transverse magnetic field, without a transport current flowing inside. This may be conceived as the limit $d/w \to \infty$ of the normalized distance between the edges of the strip and the magnets of the superconductor/soft-magnet heterostructures illustrated in figure 1, together with the limits $B_0 \to \infty$, $T \to T_0$ of the theoretical model presented in section 2.1. Here, it serves to assert the quality and performance of our tailored computational approach through an asymptotic reference to the existing analytical result.

Figure A.1 depicts the variation of the normalized hysteretic ac loss $U_{\text{ac}}/H_c^2$ with the normalized amplitude of the magnetic field $H_a/H_c$ applied to the superconductor strip in the electromagnetic steady state, using the relevant geometrical and materials data of table 1 and the ensuing value $H_c \equiv 8 \, \text{A mm}^{-1}$. Shown too is the theoretical prediction for an infinitesimally thin strip in the Bean model of the critical state [65], corresponding to the further limits $d/w \to 0$, $\sigma_0 \to \infty$. A general trait revealed for both the strip with a finite thickness and the infinitesimally thin strip is that, starting from the smallest value of $H_a/H_c$, a monotonic rise of the normalized hysteretic ac loss $U_{\text{ac}}/H_c^2$ towards a maximum occurs, followed by an asymptotically converging descent, as $H_a/H_c$ augments. Save large values of $H_a/H_c$, where the two predictions virtually meet, the normalized hysteretic ac loss $U_{\text{ac}}/H_c^2$ is always higher for the strip with a finite thickness than for the infinitesimally thin strip, the discrepancy being remarkably distinct for moderate and small values of $H_a/H_c$. Part of an explanation of this phenomenon relies on the fact that at large values of $H_a/H_c$, the critical state can form within the whole domain of the superconductor strip, rendering the geometrical effect due to the finite thickness insignificant, whereas at moderate and small values of $H_a/H_c$, the critical state can only form around the edges of the superconductor strip, letting the geometrical effect due to the

![Figure A.1](image-url). Normalized hysteretic ac loss $U_{\text{ac}}/H_c^2$ suffered by a thermo-electromagnetically isolated superconductor strip as a function of the normalized amplitude $H_a/H_c$ of an applied transverse magnetic field in the electromagnetic steady state (solid line). The analytical result corresponding to the limits $d/w \to 0$, $\sigma_0 \to \infty$ of an infinitesimally thin strip in the Bean model of the critical state (dashed line) is shown for comparison [65].
finite thickness emerge. This sensitivity of the normalized hysteretic ac loss $U_{ac}/H_{2}^2$ is analogous to previous findings for an unshielded transport current-carrying superconductor strip [66]. There exists, however, yet another important side to it which originates from our description of the critical state of the Bean model associated with a ‘smoothed’ Bean model. This sensitivity of the normalized hysteretic ac loss $U_{ac}/H_{2}^2$ generally exceeding, or at best meeting, the predictions of the latter model, the difference being particularly pronounced for moderate and small values of $H_a/H_c$.

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