Novel design of a smart magnet/superconductor heterostructure

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Abstract

Current distributions in a thin, flat superconductor ring located between two coaxial cylindrical soft magnets of high permeability are studied. Such a heterostructure protects the flux-free state of the ring even in the presence of strong total supercurrents, preventing entry of magnetic flux from the edges of the ring. A Fredholm integral equation of the first kind governing the current distribution in the Meissner state of the ring is derived and solved numerically for different relative permeabilities of the magnets and various distances between the magnets and the ring. This reveals that the current distribution tends to become homogeneous when the relative permeabilities are increased, the effect already saturating at values of these quantities of a few hundred. When the distances between the magnets and the ring are reduced, the current peaks near the circumferences of the ring decrease, thereby contributing to the large total supercurrent and, concomitantly, to the strong longitudinal component of the magnetic field.

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1. Introduction

A straight, thin superconductor strip in the flux-free, current-carrying state exhibits sharp current peaks at the edges of the strip [1]. This is why magnetic vortices may easily overcome an edge barrier and enter the strip, destroying the Meissner state already in the presence of small total transport currents or weak external magnetic fields. If, however, the current self-induced magnetic field is modified in the presence of a magnetic environment of high permeability and special geometry, the current peaks may be reduced. This shielding effect is particularly pronounced in the case where strip edges approach flat surfaces of a bulk magnet perpendicular to the plane of the strip. The flux-free state may then be protected even in the presence of currents whose strengths are comparable with that of the total current in the flux-filled critical state of the strip [2,3]. Although recently AC losses in superconducting multifilament tapes have been substantially reduced by magnetic coating of individual filaments [4–6], magnetic shielding has not, however, been used to improve critical parameters of superconductors themselves. We therefore suggest an approach for enhancing the total critical current by introducing a novel...
magnet/superconductor heterostructure which simultaneously allows the generation of high magnetic fields, and hence serves as a precursor of an electromagnet with flat superconducting coils.

2. Model

Let us consider a superconductor ring with inner radius \( R_1 \), outer radius \( R_2 \), and thickness \( 2d \), intersected symmetrically by the plane \( z = 0 \) of a cartesian system \( x, y, z \) and located between two infinitely extended, coaxial cylindrical soft magnets, as depicted in Fig. 1. The inner magnet, with relative permeability \( \mu_1 \), is supposed to occupy the space \( r \leq r_1 < R_1 \), and the outer magnet, with relative permeability \( \mu_2 \), is supposed to fill the space \( r \geq r_2 > R_2 \), adopting cylindrical polar coordinates \( (r, \varphi, z) \). Owing to the inherent rotational symmetry, the magnetic induction \( B \) possesses nonvanishing radial and longitudinal components, \( B_r \) and \( B_z \), only and can therefore be conveniently derived via \( B = \nabla \times A \) from a vector potential \( A \) with a single nonvanishing (azimuthal) component \( A \) depending on \( r \) and \( z \). If, as assumed here, \( d \ll R_1 \), variations of \( A \) across the lateral extent of the superconductor ring may be neglected and, for mathematical convenience, the ring considered to be infinitesimally thin. From Ampère’s law, the equation governing \( A \) in the region between the cylindrical magnets, \( r_1 < r < r_2 \), then takes the form

\[
\frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{1}{r^2} A = -\mu_0 s(r) \delta(z). \tag{1}
\]

where \( \mu_0 \) denotes the permeability of free space and \( s \) is the density of the circular sheet current flowing in the region of the ring, \( R_1 < r < R_2 \), the Dirac delta function confining the inhomogeneity of this equation to the plane \( z = 0 \). In the regions occupied by the magnets themselves, \( A \) obeys Eq. \( (1) \) with the right-hand side set equal to zero everywhere. Boundary conditions require that \( A \) and its weighted derivative \( (\partial A/\partial r)/\mu_1, \mu_2 \) be continuous at \( r = r_1, r_2 \); furthermore, \( A \) and \( \partial A/\partial r \) as well as \( \partial A/\partial z \) must vanish when \( r \to \infty \) or, respectively, \( z \to \pm \infty \). We seek an integral representation of the relevant component of the vector potential in terms of the density of the sheet current as a planar source, from which the magnetic induction, and hence the magnetic field, can be deduced, once the distribution of the source is known.

To this end, we first examine Eq. \( (1) \) for a circular line current of strength \( i_\rho \) flowing in a single ring of radius \( \rho \), where \( R_1 < \rho < R_2 \):

\[
\frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{1}{r^2} A = -\mu_0 i_\rho \delta(r-\rho) \delta(z). \tag{2}
\]

If \( A \) is expressed in terms of its Fourier component \( \tilde{A}_k \),

\[
A(r, z) = \frac{1}{\pi} \int_{0}^{\infty} dk \tilde{A}_k (r) \cos(kz), \tag{3}
\]

accounting for the symmetry of \( A \) with respect to \( z \), Eq. \( (2) \) transforms into

\[
\frac{d^2 \tilde{A}_k}{dr^2} + \frac{1}{r} \frac{d \tilde{A}_k}{dr} - \left( k^2 + \frac{1}{r^2} \right) \tilde{A}_k = -\mu_0 i_\rho \delta(r-\rho). \tag{4}
\]
Integration of Eq. (4) shows that the Fourier component and its first derivative must obey the relations for \( r = \rho \):

\[
\begin{align*}
\tilde{A}_k(\rho^+) - \tilde{A}_k(\rho-) &= 0, \\
\tilde{A}_k'(\rho^+) - \tilde{A}_k'(\rho-) &= -\mu_0 i_{\rho}. 
\end{align*}
\]

(5)

In addition, the requirement of continuity entails that the Fourier component and its first derivative must satisfy the conditions at \( r = r_1, r_2 \):

\[
\begin{align*}
\tilde{A}_k(r_1^+) - \tilde{A}_k(r_1^-) &= 0, \\
\tilde{A}_k'(r_1^+) - \tilde{A}_k'(r_1^-) &= 0, \\
\tilde{A}_k(r_2^-) - \tilde{A}_k(r_2^+) &= 0, \\
\tilde{A}_k'(r_2^-) - \tilde{A}_k'(r_2^+) &= 0. 
\end{align*}
\]

(6)

The general solution of Eq. (4) may be stated piecewise in the form

\[
\tilde{A}_k(r) =
\begin{cases}
\mu_0 i_{\rho} c_k^{(1)}(\rho) I_1(\rho r) ; & 0 < r < r_1, \\
\mu_0 i_{\rho} [a_k(\rho) I_1(\rho r) + b_k(\rho) K_1(\rho r)] ; & r_1 < r < \rho, \\
\mu_0 i_{\rho} [a_k(\rho) I_1(\rho r) + b_k(\rho) K_1(\rho r)] ; & \rho < r < r_2, \\
\mu_0 i_{\rho} c_k^{(2)}(\rho) K_1(\rho r) ; & r_2 < r < \infty,
\end{cases}
\]

(7)

where \( I_1 \) and \( K_1 \) denote the modified Bessel functions of the first and second kind. Subjecting Eq. (7) to the constraints, Eqs. (5) and (6), yields the coefficients

\[
c_k^{(1)}(\rho) =
(\rho / \rho_1 D(k; r_1, r_2)) \{ K_1(\rho r_1) / K_1(\rho r_2) + (1/\mu_2 - 1) \\
\times [I_1(\rho r_1) K_1(\rho r_2) - K_1(\rho r_1) I_1(\rho r_2)] K_1'(\rho r_2) \},
\]

(8)

\[
c_k^{(2)}(\rho) =
(\rho / \rho_2 D(k; r_1, r_2)) \{ I_1(\rho r_1) - I_1(\rho r_2) K_1(\rho r_2) \}
\times [K_1(\rho r_1) I_1(\rho r_2) - I_1(\rho r_1) K_1(\rho r_2)] I_1'(\rho r_2),
\]

(9)

with

\[
D(k; r_1, r_2) =
1 / 2 r_1 r_2 - (1 - 1/\mu_1) I_1'(\rho r_1) K_1(\rho r_1) / k r_2
\times [I_1(\rho r_1) K_1(\rho r_1) - I_1(\rho r_1) K_1'(\rho r_1)],
\]

(10)

and further

\[
a_k^{(1)}(\rho) = c_k^{(1)}(\rho) I_1'(\rho r_1) K_1(\rho r_1) / \mu_1
\times [I_1'(\rho r_1) K_1'(\rho r_1)],
\]

(11)

\[
b_k^{(1)}(\rho) = c_k^{(1)}(\rho) I_1(\rho r_1)(1 - 1/\mu_1) I_1(\rho r_1) I_1'(\rho r_1),
\]

(12)

\[
a_k^{(2)}(\rho) = c_k^{(2)}(\rho) K_1(\rho r_2)(1/\mu_2 - 1) K_1(\rho r_1) K_1'(\rho r_2),
\]

(13)

\[
b_k^{(2)}(\rho) = c_k^{(2)}(\rho) K_1(\rho r_2)(1 - 1/\mu_2) I_1(\rho r_1) K_1'(\rho r_2),
\]

(14)

Eqs. (7)–(14) in conjunction with Eq. (3) provide an integral representation of the relevant component of the vector potential created in the entire space by the line current preliminarily addressed. Envisaging the planar sheet current distribution to be made up of a superposition of concentric line currents, each acting as a single circular current source, the desired result for \( A \) hence ensues when replacing \( i_\rho \) by \( d\rho s(\rho) \) and integrating over the width of the ring. We thus obtain specifically in the region of the ring, \( R_1 < r < R_2 \):

\[
A(r, z) = \frac{\mu_0}{\pi} \int_0^\infty dk \left\{ \int_{R_1}^{R_2} d\rho s(\rho) \left[ a_k^{(1)}(\rho) I_1(\rho r) + b_k^{(2)}(\rho) K_1(\rho r) \right] \right\} \cos(kz).
\]

(15)

The Meissner state is characterized by the vanishing of the longitudinal component of the magnetic field inside the superconductor ring. From Eq. (15), this condition yields the following Fredholm integral equation of the first kind for the density of the circular sheet current of the ring in the region \( R_1 < r < R_2 \):

\[
\int_{R_1}^{R_2} d\rho s(\rho) M(r, \rho) = 0,
\]

(16)

with the kernel

\[
M(r, \rho) = \int_0^\infty dk k \left\{ [a_k^{(1)}(\rho) \theta(r - \rho) + a_k^{(2)}(\rho) \theta(r - \rho)] I_0(\rho r) - b_k^{(1)}(\rho) \theta(r - \rho) + b_k^{(2)}(\rho) \theta(r - \rho)] K_0(\rho r) \right\},
\]

(17)
the Heaviside theta functions singling out terms in Eq. (17) according to their dependence on \( r \) or, respectively, \( \rho \). We note that, since Eq. (16) is homogeneous, a supplementary condition must be imposed to render the determination of the sheet current distribution unique. This can be conveniently realized by keeping the total current in the superconductor ring fixed; the procedure adopted here.

3. Results and discussion

To appraise the effect of magnetic shielding on the current flow in the superconductor ring, Eq. (16) was solved numerically for a range of the geometrical and material parameters involved, and the results were displayed graphically. Fig. 2 shows the variation of the sheet current with the radial coordinate for different values of the relative permeability of the outer and the inner magnet, respectively, when the radii of the ring and those of the magnets are prescribed. A general feature due to current redistribution in the ring is the decrease of the current peak near the circumference facing that magnet, whose relative permeability is increased, and a concomitant rise of the current peak near the circumference adjacent to the opposite magnet, whose relative permeability remains unchanged. In the shielding environment of curved geometry considered here, the effect of current peak reduction is particularly pronounced at the outer circumference of the ring, unlike for the case of a straight geometry, where a symmetric reduction of current peaks with a tendency towards a homogeneous current distribution occurs [3,7]. Common to both types of geometries, however, is the evidence that this effect already saturates at moderately high values of the relative permeability.

The variation of the sheet current with the radial coordinate, when the radii of the ring and the relative permeabilities of the magnets are prescribed, is portrayed in Fig. 3 for different values of the radius of the outer and the inner magnet, respectively. Apparent is the current peak decrease near the circumference of the ring facing that magnet, whose radius approaches the respective radius of the ring, and a concomitant rise of the current peak near the circumference of the ring adjacent to the opposite magnet, whose radius remains unchanged. Again, the effect of current peak reduction is particularly pronounced at the outer circumference of the ring, resulting in a monotonic decrease of the current distribution from inside out, whereas it is less distinct at the inner circumference of the ring, leading to a more uniform current distribution, very much like for a shielding environment of straight geometry [3,7].
Entry of magnetic flux into a flat superconductor, and the consequential destruction of the Meissner state, is prevented by edge barriers of various kinds which entail some critical value of the density of the maximum edge current of the strip, \( s_b \), or by flux pinning inside the superconductor strip which acts as an extended edge barrier itself [3]. A geometrical barrier, for instance, yields the value \( s_b \approx H_{cl} \), where \( H_{cl} \) denotes the bulk lower critical magnetic field [8], and pinning gives \( s_b = 2d j_c \), where \( j_c \) means the bulk critical current density. The reduction of the edge current peaks and the transition towards homogeneous sheet current distributions exhibited in Figs. 2 and 3 leads to an increase of the total current \( i \), subject to the condition that the value of the edge current is less than \( s_b \). The magnitude of the total current in the flux-free state may be comparable with that in the critical state, \( i_c = 2(R_2 - R_1) d j_c \), providing \( s_b \geq 2d j_c \). However, even when distinct edge barriers are absent, almost the entire superconductor ring may remain flux free if the total current \( i \) is slightly less than \( i_c \) [3]. Such a situation may favour AC applications of the suggested heterostructure, anticipating electromagnetic losses will be reduced.

References
