Bistable current–voltage characteristic of a weak-link between superconducting grains

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Abstract

A model of an extended weak-link between superconducting grains, described by a nonlocal governing fluxon equation, assuming newtonian dissipation, is presented and studied analytically as well as numerically. From a ballistic trial solution based on an exact limiting form, the average macroscopic voltage across the weak-link due to a regular array of Josephson vortices moving uniformly along it is derived. Whereas the current–voltage characteristic predominantly exhibits monostability when nonlocality is weak and dissipation high, a transition to bistability and associated formation of filaments of different current densities can occur when nonlocality is strong and dissipation low.

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1. Introduction

The capability of bulk or thin film high-temperature superconductors to carry loss-free currents is essentially limited by extended defects and by weak-links between superconducting grains which often reveal Josephson junction-like traits. As an established fact, dissipation in real inhomogeneous systems appears first due to the motion of weakly pinned Josephson vortices along some easy channels between the grains where the superconductor order parameter is reduced, rather than due to the motion of strongly pinned Abrikosov vortices inside the grains [1]. This constitutes a resistive mechanism similar to that in weak-link networks representative of granular superconducting sheets [2]. Whereas for conventional Josephson junctions with their low critical current density, and hence with a Josephson penetration depth by far surpassing the London penetration depth of the material bulk, a local electrodynamic description may be adequate, for weak-links such as low-angle grain boundaries in high-quality specimens typified by a substantially increased critical current density, and hence by a Josephson penetration depth which could be much smaller than the London penetration depth of the material bulk, recourse to a nonlocal formulation must be made [3–5]. Addressing precisely the latter case, we investigate the current–voltage characteristic of a weak-link, with a regular array of Josephson junctions...
vortices moving uniformly along it, assuming newtonian dissipation, and examine the implications of nonlocal effects.

2. Model

We consider an infinitely extended, one-dimensional Josephson junction made up of two identical superconductor banks occupying the half-spaces $-\infty < x \leq -d$ and $d \leq x < \infty$ with respect to a cartesian system $x, y, z$ and adjoining a nonmagnetic, normal metallic or lossy dielectric tunnel layer of thickness $2d$ filling the space $-d < x < d$, to represent a weak-link between superconducting grains characterized by the critical current density, $j_c$. Having a driven situation in mind, we invoke a distributed transport current of density $j_x(t)$ flowing along the (positive) $x$-direction of the basic junction; a mode of operation realized when both superconductor banks contain Abrikosov vortices in the critical state such that a $z$-dependent magnetic field along the $y$-direction obtains. With the geometry of the junction addressed, the electric and magnetic fields of Josephson vortex excitations—thought to be described by the London and the Maxwell equations—have respective nonzero components $E_x$, $E_z$, and $H_y$ only, depending, in general, on coordinates $x$, $z$ and time $t$. However, owing to the parallel plate structure assumed here, for a junction of half-thickness implied to be small compared to the characteristic electrodynamic lengths of the problem at hand, $E_x$ and $H_y$ do not vary with $x$ inside the tunnel layer region, and the only nonvanishing component of the total current density is written as

$$j_x(z, t) = A|E_x|E_x + j_c \sin \varphi(z, t).$$

(1)

The first term herein specifies the density of the normal tunnel current adopting, for mathematical convenience, a field-dependent, nonohmic quasiparticle conductivity with a (positive) constant, $A$ [6], and referring to experimental observations of this kind of dependence in junctions with some homogeneously doped superconductor banks [7]; the second term reflects the density of the Josephson supercurrent which, apart from the density of the critical current, is determined by the phase difference of the superconductor order parameter across the tunnel layer, $\varphi$ [8]. From the definition of the corresponding microscopic voltage drop, $V = -2dE_x$, and the Josephson equation [9]

$$V(z, t) = -\left( \frac{\Phi_0}{2\pi} \right) \frac{\partial \varphi}{\partial t},$$

(2)

where $\Phi_0$ denotes the quantum of magnetic flux, it appears that the component $E_x$ in Eq. (1), and hence the macroscopic voltage across the junction,

$$U(z, t) = -\int_{-\infty}^{\infty} E_x(x, z, t) dx,$$

(3)

including losses in the superconductor banks as well, are related to $\partial \varphi / \partial t$.

In order to establish a governing equation for $\varphi$, we assume, according to the supposition stated before, that the half-thickness of the junction, $d$, is much smaller than the London penetration depth, $\lambda_L$; a condition certainly applying to most cases of practical significance. Furthermore, we introduce the normalized coordinate $\zeta = z / \lambda_L$, with the Josephson penetration depth [8]

$$\lambda_j = \left( \frac{\Phi_0}{4\pi\mu_0 \lambda_L j_c} \right)^{1/2},$$

(4)

where $\mu_0$ means the permeability of free space, and the normalized time $\tau = \omega_j \tau$, with the Josephson plasma frequency [8]

$$\omega_j = \left( \frac{4\pi d j_c}{\varepsilon_0 \varepsilon_0 \Phi_0} \right)^{1/2},$$

(5)

where $\varepsilon_r$ indicates the relative permittivity of the tunnel layer and $\varepsilon_0$ signifies the permittivity of free space. By way of an analysis outlined in previous work [3–5], we then arrive at the nonlinear fluxon equation in the tunnel layer region, accounting for newtonian dissipative loss,

$$\frac{\partial^2 \varphi}{\partial \tau^2} + \frac{\alpha}{2} \frac{\partial \varphi}{\partial \tau} \frac{\partial \varphi}{\partial \tau} + \sin \varphi - \gamma
\frac{1}{\pi \kappa} \int_{-\infty}^{\infty} d\xi' K_0(|\zeta - \xi'|/\nu) \frac{\partial^2 \varphi}{\partial \xi'^2},$$

(6)

with the dimensionless damping constant $\alpha = A\Phi_0/4\pi d c \nu \varepsilon_0$, the normalized transport current density $\gamma = j_t / j_c$, the dimensionless nonlocality
parameter \( \varepsilon = \lambda_L / \lambda_j \) and the modified Bessel function of the second kind and order zero, \( K_0 \).

Looking at steady-state vortex motion in the (positive) \( \zeta \)-direction of the Josephson junction of infinite extent, we first seek a ballistic solution of Eq. (6) represented by a single traveling \( 2\pi \) phase difference kink depending on the coordinate \( \chi = \zeta - ut \) alone, with slope \( d\varphi / d\chi > 0 \) throughout, for equilibrium values determined by \( \sin \varphi = \gamma \) and approached asymptotically as \( \chi \to \pm \infty \), assuming the kink velocity \( u \) is measured in units of the Swihart velocity \( v = \lambda_j \omega_j \), i.e. the maximum velocity of electromagnetic wave propagation along the junction [10]. The limit \( \varepsilon \to 0 \) of Eq. (6), when nonlocality is absent and an exact analytic solution exists (cf. [6]), suggests to set up an approximate trial solution of Eq. (6), when nonlocality is present, retaining the limiting form

\[
\varphi(\chi) = 4 \arctan(\exp(\chi/c)) + \arcsin \gamma, \tag{7}
\]

the dimensionless kink half-width, \( c \), and the dimensionless kink velocity, \( u \), being parameters defined by requiring the ansatz, Eq. (7), to satisfy Eq. (6) in an averages sense, with \( d\varphi / d\chi \) and \( d^2\varphi / dx^2 \) as respective weights. This yields the simultaneous equations

\[
u/c = (\gamma / \alpha)^{1/2} \tag{8}
\]

and

\[
u^2 + (1 - \gamma^2)^{1/2} c^2 = s(c/\varepsilon), \tag{9}
\]

where

\[
s(c/\varepsilon) = (3\pi/2)(c/\varepsilon)^3 \int_0^\infty d\lambda \sinh^2 \lambda \times \text{sech}^2((\pi c/2e) \sinh \lambda), \tag{10}
\]

from which \( c \) and \( u \) can be calculated iteratively, using the limit \( \varepsilon \to 0 \), when \( s \to 1 \), as a start. We comment that, since the critical current density of the Josephson junction must not exceed the depairing critical current density of the material bulk [11], the nonlocal regime has an upper bound of \( \varepsilon = e_{\text{max}} \approx K_{\text{GL}}^{1/2} \) with the Ginzburg–Landau parameter \( K_{\text{GL}} \), thus \( e_{\text{max}} \gg 1 \) being typical of high-temperature superconductors.

For a single Josephson vortex moving with constant velocity \( u \) along the junction in the mixed state, Eq. (2) together with Eq. (7) yields the microscopic voltage across the tunnel layer,

\[
V(\chi) = \left( \frac{\Phi_0}{\pi} \right) \left( \frac{\gamma}{\alpha} \right)^{1/2} \omega_j \text{sech}(\chi/c), \tag{11}
\]

and Eq. (3) together with Eq. (7) gives the macroscopic voltage across the junction,

\[
U(\chi) = \left( \frac{\Phi_0}{2\pi e} \right) \left( \frac{\gamma}{\alpha} \right)^{1/2} \omega_j \times \int_{-\infty}^{\infty} d\chi' \exp(-|\chi - \chi'|/\varepsilon) \text{sech}(\chi'/c). \tag{12}
\]

An extension of the preceding analysis to the situation of a regular array of Josephson vortices, having normalized nearest-neighbour spacing \( a \gg 2c \) such that interactions between individual vortices are negligible, and moving uniformly with velocity \( u \), finally seems straightforward. From Eq. (12), the average macroscopic voltage across the junction caused by this array—a directly measurable quantity—takes the simple form

\[
\langle U \rangle = \left( \frac{\Phi_0}{a} \right) u \omega_j, \tag{13}
\]

which holds in the entire nonlocal regime.

3. Results and discussion

To explore the current–voltage characteristic of the weak-link at hand, Eq. (13) in conjunction with Eqs. (8) and (9) was evaluated numerically for a range of the material parameters involved, and the results were displayed graphically. Fig. 1 shows the average macroscopic voltage as a function of the normalized transport current density, addressing various degrees of dissipation and nonlocality. A general trait due to the interplay between the normal tunnel current and the Josephson supercurrent is the decrease of the initial voltage change with increasing dissipative loss. However, whereas the voltage itself rises monotonically with the transport current density for the values of the damping constant used, when nonlocality is weak (cf. Fig. 1(a)), a nonmonotonic dependence subject to the values of the damping constant can occur, when nonlocality is strong; the average macroscopic voltage then falls across that range of the
transport current density, where a negative differential resistance appears (cf. Fig. 1(b)).

In experiments with a fixed voltage applied, rather than a uniform current impressed, characteristics of the latter type are always associated with instabilities. Thus, while for all points of the characteristic, monostability manifests itself at the highest dissipative loss, a branch point with zero slope, 0, emerges at the reduced, critical dissipative loss which, for a constant voltage maintained, splits into two stable points, 1 and 3, delineating the respective normalized current densities, $\gamma_1$ and $\gamma_3$, and encompassing an unstable point, 2, upon further reduction of the dissipative loss. This transition from monostability to bistability is not confined to the degree of nonlocality adopted here; it can occur throughout the nonlocal regime, as may easily be seen in Fig. 2. Recalling the phenomenon of current filamentation observed, e.g., in junction devices using some homogeneously doped semiconductors under external conditions of the same kind [12], we therefore predict, when bistability exists, the formation of domains carrying currents of the normalized densities $\gamma_1$ and $\gamma_3$, aligned parallel to the direction of the transport current, for the weak-link too. The assumption of steady-state requires the velocities, $u_1$ and $u_3$, together with the nearest-neighbour spacings, $a_1$ and $a_3$, distinguishing the array of Josephson vortices in the respective domains to adjust such that $u_1/a_1 = u_3/a_3$. The overall widths of these

domains, $w_1$ and $w_3$, in turn are controlled by the average density of the current flowing along the filaments,$j_f = ((w_1\gamma_1 + w_3\gamma_3)/(w_1 + w_3))j_c$. Clearly, a more accurate analysis of current filamentation would have to take the current redistributions in the superconductor banks self-consistently into account.

Fig. 2. Stability chart for the current–voltage characteristic of the weak-link between superconducting grains. The regions of monostability and bistability are separated by the critical damping constant, $\alpha = \alpha_c$, subject to the nonlocality parameter, $\varepsilon$.

References
