A novel magnet/superconductor heterostructure for high-field applications

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Abstract
The effect of magnetic shielding on the current and field distributions in a thin, flat superconductor ring located between two coaxial cylindrical soft magnets of high permeability is studied. Such a heterostructure protects the flux-free state of the ring even in the presence of strong total supercurrents, with concomitant high magnetic fields and low electromagnetic losses in ac applications. An integral equation governing the current distribution in the Meissner state of the ring is derived and solved numerically for different relative permeabilities of the magnets and various distances between the magnets and the ring. This reveals that the current distribution tends to become homogeneous when the relative permeabilities are increased, the effect already saturating at values of these quantities of the order of a few hundred. When the distances between the magnets and the ring are reduced, the current peaks near the circumferences of the ring decrease, thereby contributing to the large total supercurrent.

1. Introduction
The method of magnetic shielding of high-field superconducting devices, such as electric motors and transformers, or magnetic energy storage devices is commonly used to prevent magnetic interference with other equipment and personnel. Recently, ac losses in superconducting multifilament tapes have been substantially reduced by magnetic coating of individual filaments [1−3]. So far, magnetic shielding has not, however, been employed to improve critical parameters of superconductors themselves. In this work we suggest an approach for enhancing the total critical current of a superconducting electromagnet by considering a heterostructure made up of a superconductor ring located between two coaxial cylindrical soft magnets, which serves as a precursor of a forthcoming model of an electromagnet with flat superconducting coils. The basic idea is to exploit the geometrical effect of transport current rearrangement in magnetically shielded superconductor strips which are in the flux-free Meissner state or in the partly flux-filled critical state [4, 5].

A straight, thin superconductor strip in the flux-free, current-carrying state exhibits sharp current peaks at the edges of the strip [6]. This is why magnetic vortices may easily overcome an edge barrier and enter the strip, destroying the Meissner state already in the presence of small total transport currents or weak external magnetic fields. If, however, the current self-induced magnetic field is modified in the presence of a magnetic environment of high permeability and special geometry, the current peaks may be reduced. This effect is particularly pronounced in the case where strip edges approach flat surfaces of a bulk magnet perpendicular to the plane of the strip. The flux-free state may then be protected even in the presence of currents whose strengths are comparable with that of the total current in the flux-filled critical state of the strip [4, 5]. Utilizing the latter fact for a novel magnet/superconductor heterostructure, which simultaneously allows the generation of high magnetic fields, is the subject of the present communication.

2. Theory
We consider a superconductor ring with inner radius \(R_1\), outer radius \(R_2\), and thickness \(2d\), intersected symmetrically by the plane \(z = 0\) of a cartesian system \(x, y, z\) and located between two infinitely extended, coaxial cylindrical soft magnets, as depicted in figure 1. The inner magnet, with relative permeability \(\mu_1\), is supposed to occupy the space \(r \leq r_1 < R_1\), and the outer magnet, with relative permeability \(\mu_2\), is supposed to fill the space \(r \geq r_2 > R_2\), adopting cylindrical polar coordinates \((r, \varphi, z)\). Owing to the inherent rotational symmetry,
the magnetic induction \( B \) possesses non-vanishing radial and longitudinal components, \( B_r \) and \( B_z \), only and can therefore be conveniently derived via \( B = \nabla \times A \) from a vector potential \( A \) with a single non-vanishing (azimuthal) component \( A \) depending on \( r \) and \( z \). If, as assumed here, \( d \ll R_1 \), variations of \( A \) across the lateral extent of the superconductor ring may be neglected and, for mathematical convenience, the ring considered to be infinitesimally thin. From Amépre’s law, the equation governing \( A \) in the region between the cylindrical magnets, \( r_1 < r < r_2 \), then takes the form

\[
\frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{1}{r^2} \frac{\partial A}{\partial z^2} - 1 = -\mu_0 s(r) \delta(z)
\]

(1)

where \( \mu_0 \) denotes the permeability of free space and \( s \) is the density of the circular sheet current flowing in the region of the ring, \( r_1 < r < r_2 \), the Dirac delta function confining the inhomogeneity of equation (1) to the plane \( z = 0 \). In the regions occupied by the magnets themselves, \( A \) obeys equation (1) with the right-hand side set equal to zero everywhere. Boundary conditions require that \( A \) and its weighted derivative \((\partial A/\partial r)/r, \mu_2 \) be continuous at \( r = r_1, r_2 \); furthermore, \( A \) and \( \partial A/\partial z \) as well as \( \partial A/\partial r \) must vanish when \( r \to \infty \) or, respectively, \( z \to \pm \infty \). We seek an integral representation of the relevant component of the vector potential in terms of the density of the sheet current as a planar source, from which the magnetic induction, and hence the magnetic field, can be deduced, once the distribution of the source is known.

In order to solve equation (1), we express \( A \) in terms of its Fourier component \( \tilde{A}_k \),

\[
A(r, z) = \frac{1}{\pi} \int_0^\infty dk \tilde{A}_k(r) \cos(kz)
\]

(2)

accounting for the symmetry of \( A \) with respect to \( z \). Equation (1) then transforms into

\[
\frac{d^2 \tilde{A}_k}{dr^2} + \frac{1}{r} \frac{d \tilde{A}_k}{dr} - \left( k^2 + \frac{1}{r^2} \right) \tilde{A}_k = -\mu_0 s(r).
\]

(3)

Envisaging the planar sheet current distribution to be made up of a superposition of concentric line currents, each acting as a single circular current source, the general solution of equation (3) in the region between the cylindrical magnets, \( r_1 < r < r_2 \), may be stated piecewise in the form

\[
\tilde{A}_k(r) = \begin{cases}
\mu_0 \int_{R_1}^{R_2} d\rho s(\rho) [a_k^+(\rho) I_1(\rho r) + b_k^+(\rho) K_1(\rho r)] & r_1 < r \leq R_1 \\
\mu_0 \int_{R_1}^{R_2} d\rho s(\rho) [a_k^+ (\rho) I_1(\rho r) + b_k^+ (\rho) K_1(\rho r)] & R_1 < r < R_2 \\
\mu_0 \int_{R_1}^{R_2} d\rho s(\rho) [a_k^- (\rho) I_1(\rho r) + b_k^- (\rho) K_1(\rho r)] & R_2 \leq r < r_2
\end{cases}
\]

(4)

where \( I_1 \) and \( K_1 \) denote the modified Bessel functions of first and second kind. Consideration of the ansatz (4) for a partial line current of strength \( d\rho s(\rho) \delta(\rho - \rho') \) in the right-hand side of equation (3) shows that the coefficients \( a_k^\pm, b_k^\pm, a_k^\pm, b_k^\pm \) obey the relations

\[
\begin{align*}
a_k^+(\rho) I_1(\rho k) + b_k^+(\rho) K_1(\rho k) &= 0 \\
a_k^-(\rho) I_1(\rho k) - b_k^-(\rho) K_1(\rho k) &= 0
\end{align*}
\]

(5)

the primes indicating derivatives with respect to the argument. In the region occupied by the inner magnet, \( 0 \leq r \leq r_1 \), the regular solution of equation (3), with the right-hand side set equal to zero, may be put as

\[
\tilde{A}_k^{(1)}(r) = \mu_0 \int_{R_1}^{R_2} d\rho s(\rho) c_k^{(1)}(\rho) I_1(\rho r)
\]

(6)

and in the region filled by the outer magnet, \( r_2 \leq r < \infty \), the appropriate solution of equation (3), with the right-hand side set equal to zero, may be written as

\[
\tilde{A}_k^{(2)}(r) = \mu_0 \int_{R_1}^{R_2} d\rho s(\rho) c_k^{(2)}(\rho) K_1(\rho r)
\]

(7)

Matching the boundary conditions with respect to \( r \) gives

\[
\begin{align*}
a_k^+(\rho) &= c_k^{(1)}(\rho) k r_1 I_1'(k r_1) K_1(k r_1) / (\mu_1 - I_1(k r_1) K_1'(k r_1)) \\
b_k^+(\rho) &= c_k^{(1)}(\rho) k r_1 (1 - 1/\mu_1) I_1'(k r_1) K_1(k r_1)
\end{align*}
\]

(8)

\[
\begin{align*}
a_k^-(\rho) &= c_k^{(2)}(\rho) k r_2 (1/\mu_2 - 1) K_1'(k r_2) K_1(k r_2) \\
b_k^-(\rho) &= c_k^{(2)}(\rho) k r_2 I_1'(k r_2) K_1(k r_2) - I_1(k r_2) K_1'(k r_2) / \mu_2
\end{align*}
\]

(9)
The coefficients $c_k^{(1)}$, $c_k^{(2)}$ herein follow by subjecting expressions (8)–(11) to the constraints, equation (5), yielding

$$
c_k^{(1)}(\rho) = \frac{\rho}{kr_1} D(k; r_1, r_2)\{K_1(\alpha k)\} \left[ K_1(\alpha k) - K_1(\beta k) \right] K_0(\alpha k),
$$

$$
+ (1/\mu_2 - 1) I_1(\alpha k) K_1(\alpha k) I_1(\beta k) K_1(\alpha k) - K_1(\alpha k) I_1(\beta k) K_1(\alpha k) I_1(\beta k)\right].
$$

$$
(12)
$$

$$
c_k^{(2)}(\rho) = \frac{\rho}{kr_2} D(k; r_1, r_2)\{I_1(\alpha k)\} \left[ I_1(\alpha k) - I_1(\beta k) \right] I_0(\alpha k),
$$

$$
+ (1 - 1/\mu_1) I_1(\alpha k) K_1(\beta k) I_1(\beta k),
$$

$$
(13)
$$

with

$$
D(k; r_1, r_2) = 1/k^2 r_1 r_2 - (1 - 1/\mu_1) I_1(\alpha k) K_1(\alpha k) I_1(\beta k) K_1(\alpha k) I_1(\beta k) - K_1(\alpha k) I_1(\beta k) K_1(\alpha k) I_1(\beta k).
$$

(14)

Equations (8)–(14), in conjunction with equations (4), (6), (7) and (2), complete the desired integral representation of $A$ in the entire space. A derivation of the radial and longitudinal components of the magnetic field, $H_r$ and $H_z$, in the region between the cylindrical magnets, $r_1 < r < r_2$, gives

$$
\frac{1}{\pi} \int_0^\infty dk \int_{R_1}^{R_2} \rho^2 s(\rho)\{a_1(\rho)\} I_1(k \rho)
$$

$$
+ \rho^2 s(\rho) I_1(k \rho) \sin(k \rho),
$$

$$
r_1 < r \leq R_1
$$

$$
\frac{1}{\pi} \int_0^\infty dk \int_{R_1}^{R_2} \rho^2 s(\rho)\{a_1(\rho)\} I_1(k \rho)
$$

$$
+ \rho^2 s(\rho) I_1(k \rho) \sin(k \rho),
$$

$$
R_1 < r < R_2
$$

$$
\frac{1}{\pi} \int_0^\infty dk \int_{R_1}^{R_2} \rho^2 s(\rho)\{a_1(\rho)\} I_1(k \rho)
$$

$$
+ \rho^2 s(\rho) I_1(k \rho) \sin(k \rho),
$$

$$
R_2 \leq r < r_2
$$

(15)

and

$$
\frac{1}{\pi} \int_0^\infty dk \int_{R_1}^{R_2} \rho^2 s(\rho)\{a_1(\rho)\} I_0(k \rho)
$$

$$
- \rho^2 s(\rho) I_0(k \rho) \cos(k \rho),
$$

$$
r_1 < r \leq R_1
$$

$$
\frac{1}{\pi} \int_0^\infty dk \int_{R_1}^{R_2} \rho^2 s(\rho)\{a_1(\rho)\} I_0(k \rho)
$$

$$
- \rho^2 s(\rho) I_0(k \rho) \cos(k \rho),
$$

$$
R_1 < r < R_2
$$

$$
\frac{1}{\pi} \int_0^\infty dk \int_{R_1}^{R_2} \rho^2 s(\rho)\{a_1(\rho)\} I_0(k \rho)
$$

$$
- \rho^2 s(\rho) I_0(k \rho) \cos(k \rho),
$$

$$
R_2 \leq r < r_2
$$

(16)

The Meissner state is characterized by the vanishing of the longitudinal component of the magnetic field inside the superconductor ring. Equation (16) thus furnishes the following integral equation for the density of the circular sheet current of the ring in the region $R_1 < r < R_2$:

$$
\int_0^\infty dk \left\{ \int_{R_1}^{R_2} \rho^2 s(\rho)\{a_1(\rho)\} I_0(k \rho) - \rho^2 s(\rho) I_0(k \rho) \right\} = 0.
$$

(17)

We note the practically important limit $\mu_1, \mu_2 \to \infty$, allowing equation (17) to be written in the succinct form

$$
\int_0^\infty \frac{dk}{k^2 D(k; r_1, r_2)} \frac{d^2}{d^2 \rho} D_0(k; r_2, r) = 0
$$

(18)

with

$$
D_0(k; r_2, r) = \left[ D_0(\rho) \rho \frac{\partial}{\partial \rho} D_0(\rho) \rho \frac{\partial}{\partial \rho} D_0(k; r_2, r) \right] = 0
$$

(19)

3. Results and discussion

In a first attempt to solve equation (17), the following numerical discretization procedure was used. The region of the superconductor ring was pervaded by $n + 1$ concentric circular paths of radii $\rho_v = R_1 + v \Delta \rho$ with $v = 0, 1, \ldots, n$, where $\Delta \rho = (R_2 - R_1)/n$, carrying the respective currents $i_v = \Delta \rho s(\rho_v)/2$ and $i_n = \Delta \rho s(\rho_n)/2$ for $v = 1, 2, \ldots, n - 1$ as well as $i_n = \Delta \rho s(\rho_n)/2$. From these currents, the values of the longitudinal field component, $\vec{H}_z^{(v)} = H_z(\rho_v, i_v, z = 0)$, created along circles between these paths, with radii $\rho_v = (\rho_1 + \rho_v)/2$ for $v = 1, 2, \ldots, n$, were determined. The functional $F_z(\{i_v\}) = |\vec{H}_z^{(1)}(\{i_v\})| + |\vec{H}_z^{(2)}(\{i_v\})| + \cdots + |\vec{H}_z^{(n)}(\{i_v\})|$ was then minimized with regard to the set of currents $\{i_v\}$ so as to reach the Meissner state, where $\vec{H}_z^{(v)} = 0$ for $v = 1, 2, \ldots, n$, given the constraint that the total current, $i = i_0 + i_1 + \cdots + i_n$, is fixed. In fact, as discussed in more detail below, the Meissner state is protected if the magnitude of the edge current remains under some critical value which depends on material features and on the geometry of the edge [7]. However, it proves mathematically more convenient to explore the redistribution of a fixed total supercurrent under different circumstances and then to estimate its magnitude from the physical conditions prevailing at the edges of the ring; the position adopted here.

Figure 2 shows the variation of the sheet current with the radial coordinate for a range of values of the relative permeability of the outer and the inner magnet, respectively, when the radii of the ring and those of the magnets are prescribed. A general feature due to current redistribution in the ring is the decrease of the current peak near the circumference facing that magnet, whose relative permeability
is increased, and a concomitant rise of the current peak near the circumference adjacent to the opposite magnet, whose relative permeability remains unchanged. In the shielding environment of curved geometry considered here, the effect of current peak reduction is particularly pronounced at the outer circumference of the ring, unlike for the case of a straight geometry, where a symmetric reduction of current peaks with a tendency towards a homogeneous current distribution occurs [5, 8]. Common to both types of geometries, however, is the evidence that this effect already saturates at moderately high values of the relative permeability, equation (18) with equation (19) being representative, up to an inaccuracy of less than 1%, for values of the relative permeability greater than two hundred.

The variation of the sheet current with the radial coordinate, when the radii of the ring and the relative permeabilities of the magnets are prescribed, is portrayed in figure 3 for a range of values of the radius of the outer and the inner magnet, respectively. Apparent is the current peak decrease near the circumference of the ring facing that magnet, whose radius approaches the respective radius of the ring, and a concomitant rise of the current peak near the circumference of the ring adjacent to the opposite magnet, whose radius remains unchanged. Again, the effect of current peak reduction is particularly pronounced at the outer circumference of the ring, resulting in a monotonic decrease of the current distribution from inside out, whereas it is less distinct at the inner circumference of the ring, leading to a more uniform current distribution, very much like for a shielding environment of straight geometry [5, 8].

Entry of magnetic flux into a flat superconductor, and the consequential destruction of the Meissner state, is prevented by edge barriers of various kinds which entail some critical value of the density of the maximum edge current of the strip, \( s_b \), or by flux pinning inside the superconductor strip which acts as an extended edge barrier itself [5]. A geometrical barrier, for instance, yields the value \( s_b \cong H_{c2} \), where \( H_{c2} \) denotes the bulk lower critical magnetic field [7], and pinning gives \( s_b = 2d j_c \), where \( j_c \) means the bulk critical current density.
The reduction of the edge current peaks and the transition towards homogeneous sheet current distributions exhibited in figures 2 and 3 leads to an increase of the total current $i$, subject to the condition that the value of the edge current is less than $s_0$. The magnitude of the total current in the flux-free state may be comparable with that in the critical state, $i_c = 2(R_2 - R_1)d_j_c$, providing $s_0 \geq 2d_j_c$. However, even when distinct edge barriers are absent, almost the entire superconductor ring may remain flux free if the total current $i$ is slightly less than $i_c$ [5]. Such a situation may favour ac applications of the suggested heterostructure, anticipating electromagnetic losses will be reduced. Studies of an infinite regular stack of flat superconductor rings, extending the present investigations of a single ring to model a superconducting electromagnet with flat coils, are underway.

References
